

FAIRFIELD COUNTY MATH LEAGUE (FCML) 2011-2012

Match 5 Round 1
 Algebra I: Fractions and Exponents

- 1.) _____
- 2.) _____
- 3.) _____

1.) If $x = d^{-3}q^{-4}r^{-8}$ and $y = d^6q^{-1}r^{-1}$, then simplify and express $\left(\frac{y^{-2}}{x^6}\right)^{\frac{2}{-1}}$ in terms of d, q , and r using non-negative exponents.

2.) If a, b , and c are integers, then find all possible solutions (a, b, c) that satisfy the equation $2^a \cdot 3^{a+1} \cdot 4^{a+2} \cdot 5^{c+3} \cdot 6^{a+4} \cdot 8^{a+6} \cdot 9^{\frac{b}{2}} \cdot 10^{a+8} = 100$

3.) If $(12^n \cdot (-2)^n \cdot 4^x \cdot 9^{x+6})^n = 1$, and x and n are integers, then find all real values of $\frac{n}{x}$.

FAIRFIELD COUNTY MATH LEAGUE (FCML) 2011-2012

Match 5 Round 2
Algebra I: Fractional Expressions
And Equations

1.) _____

2.) _____

3.) _____

1.) Simplify as a fully factored, singular fraction: $\frac{x}{\frac{1}{x} + \frac{1}{1}} + \frac{\frac{1}{x} - \frac{1}{x}}{\frac{1}{x} + 1}$

2.) Solve for x: $\frac{-4}{6} - \frac{9+x}{2x-7} = \frac{(x+9)(7-2x)}{2x^2-25x+6}$

3.) Find all real values of x such that $\frac{x^2-2}{-1+\frac{1}{x}} = \frac{\frac{1+x}{2} - \frac{1+x}{2}}{x}$.

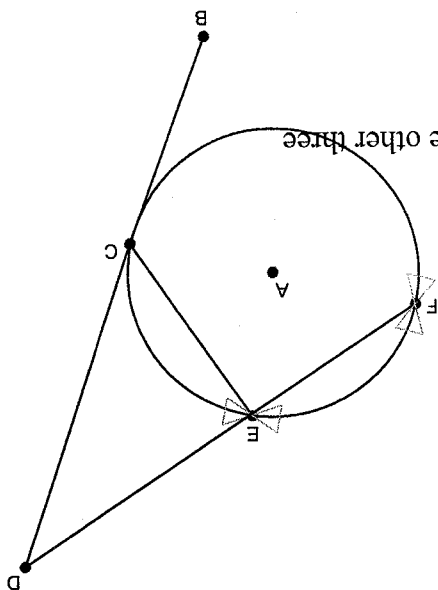
FAIRFIELD COUNTY MATH LEAGUE (FCML) 2011-2012

Match 5 Round 3
 Geometry: Circles

Note: Diagrams are not to scale.

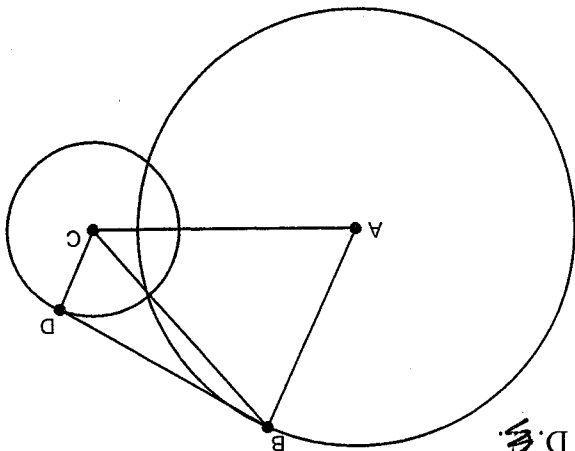
- 1.) _____
- 2.) _____
- 3.) _____

1.) Segment BD is tangent to circle A at B . If $\overline{FE} \perp \overline{CE}$ and $m\widehat{FE} = 44^\circ$, then find the measure of $\angle D$.



2.) Four circles of radius 2 are internally tangent to a larger circle and are each of the four are tangent to 2 of the other three circles. Find the radius of the large circle.

3.) Segment BD is tangent to circle A at B and AC is tangent to circle C at C . If $CD = 2$, $AB = 3$, and $AC = 4$, then find BC .



FAIRFIELD COUNTY MATH LEAGUE (FCML) 2011-2012

Match 5 Round 4
Algebra II: Quadratic Equations
and Complex Numbers

- 1.) _____
- 2.) _____
- 3.) _____

1.) Find the distance between the points representing $1 + 6i$ and $-1 - 4i$ in the complex plane.

2.) The quadratic equation $x^2 + bx + c = 0$ (where b and c are complex numbers) has roots $9 - 4i$ and $-9 + 6i$. Find the product bc .

3.) For what real values of k does the equation $x^2 + 2(k + 1)x + 4 = 0$ have roots that are not real?

FAIRFIELD COUNTY MATH LEAGUE (FCML) 2011-2012

Match 5 Round 5
 Trigonometry: Solving Trigonometric Equations

1.) _____

2.) _____

3.) _____

1.) Solve for all real values of x , given that $0 \leq x < 2\pi$: $\sin\left(\frac{1}{2}x\right) = \frac{1}{2}$

2.) Solve for all real values of x , given that $-\pi \leq x < \pi$: $\cos\left(2x + \frac{\pi}{2}\right) = \sin x$

3.) Solve for all real values of x : $\left(\sin^{-1}\left(\frac{x}{4}\right)\right)^2 = \frac{\pi^2}{16}$

FAIRFIELD COUNTY MATH LEAGUE (FCML) 2011-2012

Match 5 Round 6

Discrete Math: Sequences and Series

1.) _____

2.) _____

3.) _____

1.) Evaluate the sum $\sum_{n=6}^H (n-1)(n+1)$

2.) The first four terms of an arithmetic series add up to 10. The first seven terms of the same series add up to -14. Find the 9th term of the series.

3.) A sequence is defined as $s_n = an^2 + bn + c$. If the first term of the sequence is 7, the second term is 5, and the third term is 1, then find the product $a \cdot b \cdot c$.

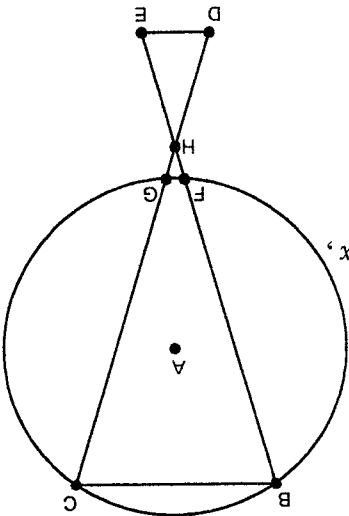
FAIRFIELD COUNTY MATH LEAGUE (FCML) 2011-2012

Match 5 Team Round

- 1.) Solve for all values of x that make the equation $2^{3^x} = \left(\frac{4}{3}\right)^x$ true.

- 2.) Find all real values of y such that the following equation has no solution:

$$1 + \frac{x+8}{x-5} = \frac{2x^2 - x - 4}{(x-2y)(x-5)}$$



- 3.) In circle A , $CG = 7$, $GH = 3$, $HD = 4$, $FH = \frac{5}{2}$, $BF = x$, and $HE = y$. Find all ordered pairs (x, y) such that $\triangle HBC \sim \triangle HED$.

- 4.) If a , and b are complex numbers, then find all possible ordered pairs (a, b) that are solutions to the system of equations:
 $8a + 8bi = -3 - 8i$
 $8ai + 4b = 7 - 3i$

- 5.) Solve for all real values of x , given that $0 \leq x < 2\pi$: $3 \sec^2 x - 6 \tan^2 x = 2\sqrt{3} \tan x$

- 6.) Evaluate $\sum_{j=3}^4 \left(\sum_{i=2}^j \left(\sum_{n=1}^i \left(\sum_{k=1}^n \left(\frac{1}{k} \right) \right) \right) \right)$

KEY

FAIRFIELD COUNTY MATH LEAGUE (FCML) 2011-2012

Match 5 Round 1
Algebra I: Fractions and Exponents

- 1.) $p^3 q^{18} r^{25}$
- 2.) $(-4, -4, -5)$
- 3.) $-\frac{3}{2}$

1.) If $x = p^{-3} q^{-4} r^{-8}$ and $y = p^6 q^{-1} r^{-1}$, then simplify and express $\left(\frac{y^{-2}}{x^6}\right)^{\frac{2}{-1}}$ in terms of p, q, r using non-negative exponents.

2.) If a, b, c are integers, then find all possible solutions (a, b, c) that satisfy the equation $2^a \cdot 3^{a+1} \cdot 4^{a+2} \cdot 5^{c+3} \cdot 6^{a+4} \cdot 8^{a+6} \cdot 9^{\frac{b}{2}} \cdot 10^{a+8} = 100$

3.) If $(12^n \cdot (-2)^n \cdot 4^x \cdot 9^{x+6})^n = 1$, and x and n are integers, then find all real values of $\frac{n}{x}$.

FAIRFIELD COUNTY MATH LEAGUE (FCML) 2011-2012

Match 5 Round 2
Algebra I: Fractional Expressions
And Equations

- 1.) $\frac{x(2x-1)}{(x+1)(x-1)}$
- 2.) $20 \text{ or } -\frac{1}{2}$
- 3.) $\sqrt[3]{2}$

1.) Simplify as a fully factored, singular fraction: $\frac{x}{1} + \frac{1}{x} - \frac{1}{x+1}$

2.) Solve for x: $\frac{-4}{6} - \frac{9+x}{2x-7} = \frac{(x+9)(7-2x)}{2x^2-25x+6}$

3.) Find all real values of x such that $\frac{x^2-2}{-1+\frac{1}{x}} = \frac{\frac{1+x}{2} - \frac{1+x}{2}}{x}$

K3Y

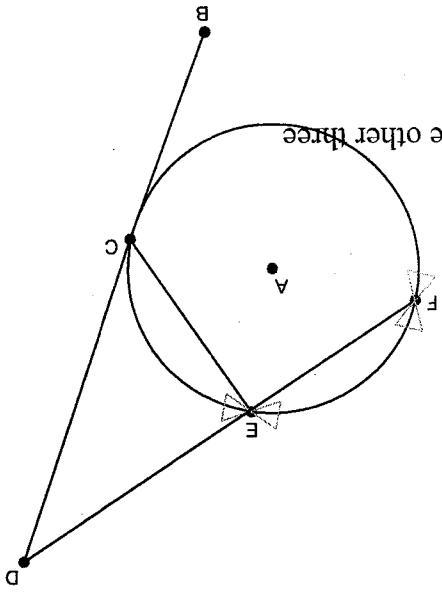
FAIRFIELD COUNTY MATH LEAGUE (FCML) 2011-2012

Match 5 Round 3
Geometry: Circles

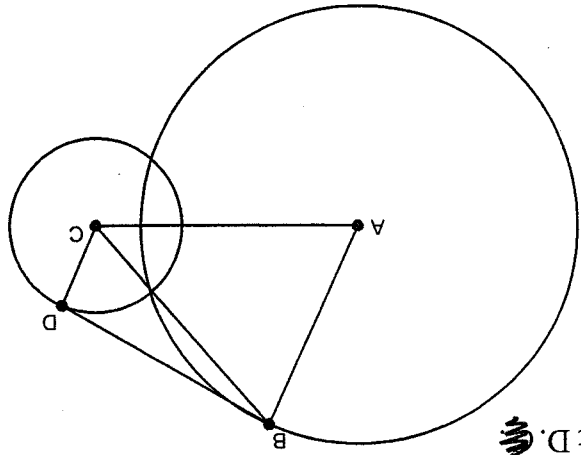
Note: Diagrams are not to scale.

- 1.) 22°
- 2.) $2 + 2\sqrt{2}$
- 3.) $\sqrt{19}$

1.) Segment BD is tangent to circle A at B . If $\overline{FE} \perp \overline{CE}$ and $m\widehat{FE} = 44^\circ$, then find the measure of $\angle D$.



2.) Four circles of radius 2 are internally tangent to a larger circle and are each of the four are tangent to 2 of the other three circles. Find the radius of the large circle.



3.) Segment BD is tangent to circle A at B and CD is tangent to circle A at C . If $CD = 2$, $AB = 3$, and $AC = 4$, then find BC .

KEY

KEY

FAIRFIELD COUNTY MATH LEAGUE (FCML) 2011-2012

Match 5 Round 4
Algebra II: Quadratic Equations
and Complex Numbers

- 1.) $2\sqrt{26}$
 - 2.) $180 + 114i$
 - 3.) $-3 < k < 1$
accept $(-3, 1)$
- 1.) Find the distance between the points representing $1 + 6i$ and $-1 - 4i$ in the complex plane.

2.) The quadratic equation $x^2 + bx + c = 0$ (where b and c are complex numbers) has roots $9 - 4i$ and $-9 + 6i$. Find the product bc .

3.) For what real values of k does the equation $x^2 + 2(k+1)x + 4 = 0$ have roots that are not real?

FAIRFIELD COUNTY MATH LEAGUE (FCML) 2011-2012

Match 5 Round 5
Trigonometry: Solving Trigonometric Equations

1.) $\sin \frac{\pi}{3}, \frac{5\pi}{3}$ ~~$\frac{\pi}{3}, \frac{5\pi}{3}$~~

2.) $-\pi, -\frac{2\pi}{3}, \frac{2\pi}{3}, 0, \frac{\pi}{3}$

3.) $\pm 2\sqrt{2}$

1.) Solve for all real values of x , given that $0 \leq x < 2\pi$: $\sin\left(\frac{1}{2}x\right) = \frac{1}{2}$

2.) Solve for all real values of x , given that $-\pi \leq x < \pi$: $\cos\left(2x + \frac{\pi}{2}\right) = \sin x$

3.) Solve for all real values of x : $\left(\sin^{-1}\left(\frac{x}{4}\right)\right)^2 = \frac{16}{\pi^2}$

KEY

FAIRFIELD COUNTY MATH LEAGUE (FCML) 2011-2012

Match 5 Round 6

Discrete Math: Sequences and Series

1.)

445

2.)

-17

3.)

-7

$$1.) \text{ Evaluate the sum } \sum_{n=1}^6 (n-1)(n+1)$$

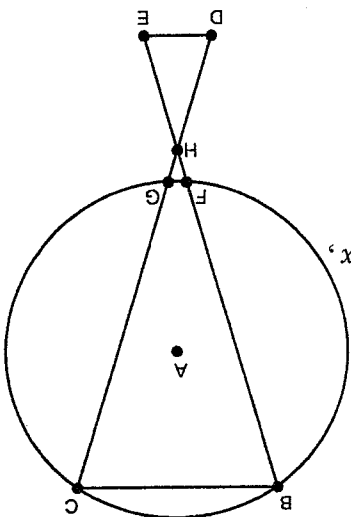
2.) The first four terms of an arithmetic series add up to 10. The first seven terms of the same series add up to -14. Find the 9th term of the series.

3.) A sequence is defined as $s_n = an^2 + bn + c$. If the first term of the sequence is 7, the second term is 5, and the third term is 1, then find the product $a \cdot b \cdot c$.

KEY

$$\begin{array}{r} 7 \\ 3 \overline{) 21} \\ \underline{21} \\ 0 \end{array}$$
$$\frac{(5-x)(27-x)}{2x^2-x-4} = \frac{x+8}{x-5} + 1$$

$$y \neq 1, \frac{12}{5}, \frac{13}{2}, \frac{4}{-3}, \frac{4}{1+\sqrt{3}}$$

$$\Delta H_{BC} \sim \Delta H_{ED}.$$


solutions to the system of equations:

$$8a + 8bi = -3 - 8i$$

$$8ai + 4b = 7 - 3i$$

$$\left(-\frac{3}{8}, -\frac{11}{12}, -\frac{1}{12}\right)$$

5.) Solve for all real values of x , given that $0 \leq x < 2\pi$: $3 \sec^2 x - 6 \tan^2 x = 2\sqrt{3} \tan x$

$$\frac{9}{11}, \frac{9}{17}, \frac{9}{27}, \frac{9}{55}$$

6.) Evaluate $\sum_{j=3}^4 \left(\sum_{i=2}^j \left(\sum_{l=1}^n \binom{n}{l} \right) \right)$

$$\begin{array}{r} 4 \\ \hline 35 \end{array}$$